

A SPECIAL CLASS OF UNIVALENT OPERATORS

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ABSTRACT. Let $H(U)$ be the space of analytic functions in the unit disk U and let $h \in H(U)$. We define a subset $\mathcal{K}_{(\beta, \gamma), h} \subset H(U)$ such that the operator $A_{(\beta, \gamma), h} : \mathcal{K}_{(\beta, \gamma), h} \rightarrow H(U)$ given by

$$A_{(\beta, \gamma), h}(f)(z) = \left[\frac{\beta + \gamma}{h^\gamma(z)} \int_0^z f^\beta(t) h^{\gamma-1}(t) h'(t) dt \right]^{1/\beta}$$

is well defined. Then we determine a class of functions whose images by $A_{(\beta, \gamma), h}$ operators are univalent. In addition, we give some particular cases of our main result obtained for appropriate choices of h, β and γ .

1. INTRODUCTION

Let $H(U)$ be the space of all analytic functions in the unit disk $U = \{z \in \mathbf{C} : |z| < 1\}$ and let $h \in \mathcal{A} = \{h \in H(U) : h(0) = 0, h'(0) \neq 0, h(z)h'(z) \neq 0, \text{ for } 0 < |z| < 1\}$. For $f \in \mathcal{K}_{(\beta, \gamma), h} \subset H(U)$ let $F = A_{(\beta, \gamma), h}(f)$ where

$$(1.1) \quad F(z) = \left[\frac{\beta + \gamma}{h^\gamma(z)} \int_0^z f^\beta(t) h^{\gamma-1}(t) h'(t) dt \right]^{1/\beta}, \quad \beta, \gamma \in \mathbf{C}.$$

This type of integral operators and different particular cases were studied in several papers like [1], [2], [3], [6], [7], [8] and others.

In the present paper, first we will determine sufficient conditions on h and the correspondent classes $\mathcal{K}_{(\beta, \gamma), h}$ such that the operator given by (1.1) will be well defined. Then we will find a class of functions whose images by $A_{(\beta, \gamma), h}$ operator are univalent in U and in addition some particular cases obtained for different choices of h, β and γ will be given.

2. PRELIMINARIES

In order to prove our main results, we will need the following definitions and lemmas presented in this section.

1991 *Mathematics Subject Classification.* Primary 30C80; Secondary 30C45, 30E20.

Key words and phrases. Integral operator, univalent function, differential subordination, subordination chain, starlike function.

Like in [8], let $c \in \mathbf{C}$ with $\operatorname{Re} c > 0$ and let $N = N(c) = \frac{|c|\sqrt{1+2\operatorname{Re} c} + \operatorname{Im} c}{\operatorname{Re} c}$.

Considering the univalent function $k(z) = \frac{2Nz}{1-z^2}$, we define the "open door" function

$$(2.1) \quad R_c(z) = k\left(\frac{z+b}{1+bz}\right), \quad z \in U.$$

Note that R_c is univalent in U , $R_c(0) = c$ and $R_c(U) = k(U)$ is the complex plane slit along the half lines $\operatorname{Re} w = 0, \operatorname{Im} w \geq N$ and $\operatorname{Re} w = 0, \operatorname{Im} w \leq -N$.

For $f, g \in H(U)$ we say that f is subordinate to g , written $f(z) \prec g(z)$, if g is univalent in U , $f(0) = g(0)$ and $f(U) \subset g(U)$.

We denote by $A = \{f \in H(U) : f(0) = f'(0) - 1 = 0\}$ and let $D = \{\phi \in H(U) : \phi(z) \neq 0 \text{ for } z \in U, \phi(0) = 1\}$.

Lemma 1. [4] Let $\phi, \Phi \in D$ and let $\alpha, \beta, \gamma, \delta \in \mathbf{C}$ with $\beta \neq 0, \alpha + \delta = \beta + \gamma$ and $\operatorname{Re}(\alpha + \delta) > 0$. If $f \in A$ satisfies

$$\alpha \frac{zf'(z)}{f(z)} + \frac{z\phi'(z)}{\phi(z)} + \delta \prec R_{\alpha+\delta}(z),$$

where R_c is defined by (2.1) and if the function F is defined by

$$(2.2) \quad F = A_{\beta, \gamma}(f) \text{ where } A_{\beta, \gamma}(f)(z) = \left[\frac{\beta + \gamma}{z^\gamma} \int_0^z f^\beta(t) t^{\gamma-1} dt \right]^{1/\beta}$$

then

$$F \in A, \quad \frac{F(z)}{z} \neq 0, \quad z \in U \text{ and } \operatorname{Re} \left[\beta \frac{zF'(z)}{F(z)} + \frac{z\Phi'(z)}{\Phi(z)} + \gamma \right] > 0, \quad z \in U.$$

(All powers in (2.2) are principal ones.)

A function $f \in A$ is called a starlike function of order α , $\alpha < 1$, if $\operatorname{Re} \frac{zf'(z)}{f(z)} > \alpha$ for all $z \in U$ and we denote by $S^*(\alpha)$ the class of all these functions.

Lemma 2. [9] Let $\beta > 0, \beta + \gamma > 0$ and consider the integral operator $A_{\beta, \gamma}$ defined by (2.2). If $\alpha \in \left[-\frac{\gamma}{\beta}, 1\right)$, then the order of starlikeness of the class $A_{\beta, \gamma}(S^*(\alpha))$, i.e. the largest number $\delta = \delta(\alpha; \beta, \gamma)$ such that $A_{\beta, \gamma}(S^*(\alpha)) \subset S^*(\delta)$ is given by $\delta(\alpha; \beta, \gamma) = \inf\{\operatorname{Re} q(z) : z \in U\}$, where

$$q(z) = \frac{1}{\beta Q(z)} - \frac{\gamma}{\beta} \quad \text{and} \quad Q(z) = \int_0^1 \left(\frac{1-z}{1-tz} \right)^{2\beta(1-\alpha)} t^{\beta+\gamma-1} dt.$$

Moreover if $\alpha \in [\alpha_0, 1)$, where $\alpha_0 = \max \left\{ \frac{\beta - \gamma - 1}{2\beta}; -\frac{\gamma}{\beta} \right\}$ and $g = A_{\beta, \gamma}(f)$ where $f \in S^*(\alpha)$, then

$$\operatorname{Re} \frac{zg'(z)}{g(z)} > \delta(\alpha; \beta, \gamma) = \frac{1}{\beta} \left[\frac{\beta + \gamma}{{}_2F_1(1, 2\beta(1 - \alpha), \beta + \gamma + 1; \frac{1}{2})} - \gamma \right], \quad z \in U,$$

where ${}_2F_1$ represents the hypergeometric function.

The next lemma concerns subordination (or Loewner) chains. A function $L(z; t)$, $z \in U$, $t \geq 0$ is called a *subordination chain* if $L(\cdot; t)$ is analytic and univalent in U for all $t \geq 0$, $L(z; \cdot)$ is continuously differentiable on $[0, +\infty)$ for all $z \in U$ and $L(z; s) \prec L(z; t)$ when $0 \leq s \leq t$ [10, p. 157].

Lemma 3. [10, p. 159] *The function $L(z; t) = a_1(t)z + \dots$ with $a_1(t) \neq 0$ for all $t \geq 0$ and $\lim_{t \rightarrow +\infty} |a_1(t)| = +\infty$ is a subordination chain if and only if*

$$\operatorname{Re} \left[z \frac{\partial L / \partial z}{\partial L / \partial t} \right] > 0, \quad z \in U, \quad t \geq 0.$$

The next lemma is a slight modification of a result of K. Sakaguchi which provides a sufficient condition for univalence.

Lemma 4. [11, Corollary 3] *Let $\operatorname{Re} \beta > -\frac{1}{2}$ and for $F \in H(U)$, with $F'(0) \neq 0$ let*

$$(2.3) \quad J(\beta, F)(z) = (\beta - 1) \frac{zF'(z)}{F(z)} + \frac{zF''(z)}{F'(z)} + 1.$$

If $\operatorname{Re} J(1, F)(z) > -\frac{1}{2}$, $z \in U$ or $\operatorname{Re} J(\beta, F)(z) > -\frac{1}{2}$, $z \in U$ when $\beta \neq 1$ and $F(0) = 0$, then F is univalent in U .

The last lemma deals with the univalent solutions of *Briot-Bouquet differential equations* and represents a simplified form of Theorem 1 from [5].

Lemma 5. [5, Theorem 1] *Let $\beta, \gamma \in \mathbb{C}$ with $\beta \neq 0$ and let $h(z) = c + h_1z + \dots$ be analytic in U . If $\operatorname{Re} [\beta h(z) + \gamma] > 0$, $z \in U$ then the solution of the differential equation*

$$q(z) + \frac{zq'(z)}{\beta q(z) + \gamma} = h(z), \quad \text{with } q(0) = c,$$

is analytic in U and the solution satisfies $\operatorname{Re} [\beta q(z) + \gamma] > 0$, $z \in U$.

3. MAIN RESULTS

Our first result gives us sufficient conditions on h function such that the integral operator (1.1) is well defined.

Theorem 1. Let $\beta, \gamma \in \mathbf{C}$ with $\operatorname{Re}(\beta + \gamma) > 0$ and let $h \in \mathcal{A}$. Then the integral operator given by (1.1) is well defined on the subset

$$\mathcal{K}_{(\beta, \gamma), h} = \left\{ f \in H(U) : f(0) = 0, f'(0) \neq 0, \beta \frac{zf'(z)}{f(z)} + J(\gamma, h)(z) \prec R_{\beta+\gamma}(z) \right\}.$$

Proof. In order to prove our theorem we will use Lemma 1 for $\alpha := \beta$ and $\gamma := \delta$. Taking in this lemma $\phi(z) = \left(\frac{h(z)}{z}\right)^{\gamma-1} h'(z)$ and $\Phi(z) = \left(\frac{h(z)}{z}\right)^{\gamma}$, since $h \in \mathcal{A}$ we easily deduce $\phi, \Phi \in D$.

A simple computation shows that $\frac{z\phi'(z)}{\phi(z)} = J(\gamma, h)(z) - \gamma$ hence the condition

$$\begin{aligned} \beta \frac{zf'(z)}{f(z)} + \frac{z\phi'(z)}{\phi(z)} + \gamma \prec R_{\beta+\gamma}(z) \quad \text{is equivalent to} \\ \beta \frac{zf'(z)}{f(z)} + J(\gamma, h)(z) \prec R_{\beta+\gamma}(z) \end{aligned}$$

i.e. $f \in \mathcal{K}_{(\beta, \gamma), h}$ and using Lemma 1 we deduce that the function $F = A_{(\beta, \gamma), h}(f)$ is analytic in U . \square

Theorem 2. Let $\beta, \gamma \in \mathbf{C}$ with $\beta + \gamma > 0$. For a function $h \in \mathcal{A}$ we denote by

$$m = \inf \left\{ \operatorname{Re} \gamma \frac{zh'(z)}{h(z)} : z \in U \right\} \quad \text{and by} \quad M = \sup \left\{ \operatorname{Re} \gamma \frac{zh'(z)}{h(z)} : z \in U \right\}.$$

Let δ be a real number such that

$$(3.1) \quad m - (\beta + \gamma) < \delta \leq \min \left\{ m; m - \frac{\beta + \gamma - 1}{2} \right\}$$

and

$$(3.2) \quad \max\{0; M\} \leq \frac{\beta + \gamma}{{}_2F_1(1, 2(\beta + \gamma + \delta - m), \beta + \gamma + 1; \frac{1}{2})}$$

and suppose that h function satisfies the inequality

$$(3.3) \quad \operatorname{Re} J(-\gamma, h)(z) \geq -\frac{1}{2} - \frac{\beta + \gamma}{{}_2F_1(1, 2(\beta + \gamma + \delta - m), \beta + \gamma + 1; \frac{1}{2})}, \quad z \in U.$$

If $f \in \mathcal{K}_{(\beta, \gamma), h}$ and $\operatorname{Re} J(\beta, f)(z) > -\delta$, $z \in U$, then $F = A_{(\beta, \gamma), h}(f)$ given by (1.1) is univalent in U . In addition f is also univalent in U .

Proof. Since $f \in \mathcal{K}_{(\beta, \gamma), h}$, using Theorem 1 we have that $F = A_{(\beta, \gamma), h}(f)$ is analytic in U and by (1.1) we deduce

$$(3.4) \quad f(z) = \frac{1}{(\beta + \gamma)^{1/\beta}} F(z) \left[\beta \frac{h(z)}{h'(z)} \frac{F'(z)}{F(z)} + \gamma \right]^{1/\beta}.$$

Letting

$$(3.5) \quad L(z; t) = \frac{1}{(\beta + \gamma)^{1/\beta}} F(z) \left[(1+t)\beta \frac{h(z)}{h'(z)} \frac{F'(z)}{F(z)} + \gamma \right]^{1/\beta}, \quad t \geq 0$$

we will prove that $L(z; t)$ is a subordination chain. Using (3.5), a simple computation shows

$$(3.6) \quad z \frac{\partial L / \partial z}{\partial L / \partial t} = (1+t)T(z) + \gamma \frac{zh'(z)}{h(z)}$$

where

$$(3.7) \quad T(z) = J(\beta, F)(z) + \frac{zh'(z)}{h(z)} - \frac{zh''(z)}{h'(z)} - 1.$$

According to Lemma 3 and using (3.5), to prove that $L(z; t)$ is a subordination chain it is sufficient to show the next two inequalities:

$$(3.8) \quad \operatorname{Re} T(z) > 0, \quad z \in U \quad \text{and} \quad \operatorname{Re} \left\{ T(z) + \gamma \frac{zh'(z)}{h(z)} \right\} > 0, \quad z \in U.$$

From $f'(0) \neq 0$ we have $F'(0) \neq 0$ and if $L(z; t) = a_1(t)z + \dots$ we get

$$\lim_{t \rightarrow +\infty} |a_1(t)| = \lim_{t \rightarrow +\infty} \left| \frac{\partial L(0; t)}{\partial z} \right| = \lim_{t \rightarrow +\infty} \left| \frac{F'(0)}{(\beta + \gamma)^{1/\beta}} (t\beta + \beta + \gamma)^{1/\beta} \right| = +\infty.$$

Since

$$J(\beta, f)(z) = p(z) + \frac{zp'(z)}{p(z)} - \gamma \frac{zh'(z)}{h(z)} \quad \text{where} \quad p(z) = T(z) + \gamma \frac{zh'(z)}{h(z)},$$

using the assumption $\operatorname{Re} J(\beta, f)(z) > -\delta, z \in U$, we deduce

$$(3.9) \quad \operatorname{Re} \left\{ p(z) + \frac{zp'(z)}{p(z)} \right\} > -\delta + m, \quad z \in U.$$

Considering the differential equation

$$p(z) + \frac{zp'(z)}{p(z)} = J(\beta, f)(z) + \gamma \frac{zh'(z)}{h(z)},$$

from (3.1) we have

$$\operatorname{Re} \left\{ J(\beta, f)(z) + \gamma \frac{zh'(z)}{h(z)} \right\} > -\delta + m \geq 0, \quad z \in U$$

and by Lemma 5 we conclude that this equation has an analytic solution in U .

Denoting by $q(z) = \frac{p(z)}{\beta + \gamma}$, then $q(0) = 1$ and from (3.9) we obtain

$$(3.10) \quad \operatorname{Re} \left\{ q(z) + \frac{zq'(z)}{(\beta + \gamma)q(z)} \right\} > \frac{m - \delta}{\beta + \gamma}, \quad z \in U.$$

Since (3.10) holds, by using Lemma 2 for $\beta := \beta + \gamma$, $\gamma := 0$, $\alpha := \frac{m - \delta}{\beta + \gamma}$ we obtain

$$\operatorname{Re} q(z) > \frac{1}{{}_2F_1(1, 2(\beta + \gamma + \delta - m), \beta + \gamma + 1; \frac{1}{2})}, z \in U$$

or

$$(3.11) \quad \operatorname{Re} p(z) > \frac{\beta + \gamma}{{}_2F_1(1, 2(\beta + \gamma + \delta - m), \beta + \gamma + 1; \frac{1}{2})}, z \in U.$$

From (3.11) and (3.2) we deduce

$$\begin{aligned} \operatorname{Re} T(z) &> \frac{\beta + \gamma}{{}_2F_1(1, 2(\beta + \gamma + \delta - m), \beta + \gamma + 1; \frac{1}{2})} - \operatorname{Re} \gamma \frac{zh'(z)}{h(z)} > \\ &> \frac{\beta + \gamma}{{}_2F_1(1, 2(\beta + \gamma + \delta - m), \beta + \gamma + 1; \frac{1}{2})} - M \geq 0, z \in U, \end{aligned}$$

hence $\operatorname{Re} T(z) > 0$, $z \in U$.

Similarly, from (3.11) and (3.2) we have

$$(3.12) \quad \operatorname{Re} \left\{ T(z) + \gamma \frac{zh'(z)}{h(z)} \right\} > \frac{\beta + \gamma}{{}_2F_1(1, 2(\beta + \gamma + \delta - m), \beta + \gamma + 1; \frac{1}{2})} > 0, z \in U$$

hence both conditions of (3.8) are satisfied and according to Lemma 3, the function $L(z; t)$ is a subordination chain; thus $f(z) = L(z; 0)$ is univalent in U .

Using (3.7) we obtain

$$J(\beta, F)(z) = T(z) + \frac{zh''(z)}{h'(z)} + 1 - \frac{zh'(z)}{h(z)} = T(z) + \gamma \frac{zh'(z)}{h(z)} + J(-\gamma, h)(z)$$

and by combining this equality with (3.12) and (3.3) we get

$$\begin{aligned} \operatorname{Re} J(\beta, F)(z) &> \frac{\beta + \gamma}{{}_2F_1(1, 2(\beta + \gamma + \delta - m), \beta + \gamma + 1; \frac{1}{2})} \\ &+ \operatorname{Re} J(-\gamma, h)(z) \geq -\frac{1}{2}, z \in U \end{aligned}$$

hence by Lemma 4, the function F is univalent in U , which completes the proof of the theorem. \square

4. PARTICULAR CASES

1. Taking $h(z) = ze^{\lambda z}$, $\lambda \leq 1$ in Theorem 2, for the case $\gamma \in \mathbf{R}$ we have $m = \gamma - |\gamma||\lambda|$ and $M = \gamma + |\gamma||\lambda|$. Then (3.3) becomes

$$(4.1) \quad \operatorname{Re} \left[1 - \gamma(1 + \lambda z) - \frac{1}{1 + \lambda z} \right] \geq -\frac{1}{2} - \frac{\beta + \gamma}{{}_2F_1(1, 2(\beta + \gamma + \delta - m), \beta + \gamma + 1; \frac{1}{2})}, \quad z \in U.$$

Let consider the case $\beta + \gamma \geq 1$ and let $\delta := \gamma - |\gamma||\lambda| - \frac{\beta + \gamma - 1}{2}$. Then ${}_2F_1(1, 2(\beta + \gamma + \delta - m), \beta + \gamma + 1; \frac{1}{2}) = 2$ and (4.1) is equivalent to

$$(4.2) \quad \operatorname{Re} \left[-\gamma\lambda z - \frac{1}{1 + \lambda z} \right] \geq -\frac{3}{2} - \frac{\beta - \gamma}{2}, \quad z \in U.$$

Since

$$\operatorname{Re} \left[-\gamma\lambda z - \frac{1}{1 + \lambda z} \right] > -|\gamma||\lambda| - \frac{1}{1 - |\lambda|}, \quad z \in U,$$

we deduce that (4.2) holds if $2|\gamma||\lambda|^2 - (\beta - \gamma + 2|\gamma| + 3)|\lambda| + \beta - \gamma + 1 \geq 0$ and using Theorem 2 we obtain:

Corollary 1. *Let $\beta, \gamma \in \mathbf{R}$ with $\beta + \gamma \geq 1$ and let $\lambda \in \mathbf{C}$ with $|\lambda| \leq 1$ such that*

$$2|\gamma||\lambda| \leq \beta - \gamma$$

and

$$2|\gamma||\lambda|^2 - (\beta - \gamma + 2|\gamma| + 3)|\lambda| + \beta - \gamma + 1 \geq 0.$$

If $f \in H(U)$ with $f(0) = 0$, $f'(0) \neq 0$ and

$$\beta \frac{zf'(z)}{f(z)} + 1 + \gamma(1 + \lambda z) - \frac{1}{1 + \lambda z} \prec R_{\beta + \gamma}(z) \text{ satisfies}$$

$$\operatorname{Re} J(\beta, f)(z) > |\gamma||\lambda| + \frac{\beta - \gamma - 1}{2}, \quad z \in U$$

then $F = A_{(\beta, \gamma), ze^{\lambda z}}(f)$ is univalent in U ; in addition f is also univalent in U .

2. Taking $h(z) = \frac{z}{1 + \lambda z}$, $\lambda \leq 1$ in Theorem 2, a simple calculus shows that if $\gamma \geq 0$ then $m = \frac{\gamma}{1 + |\lambda|}$ and $M = \frac{\gamma}{1 - |\lambda|}$.

Let consider the case $\beta + \gamma \geq 1$ and let $\delta := \frac{\gamma}{1 + |\lambda|} - \frac{\beta + \gamma - 1}{2}$. By similar reasons to the first particular case, (3.3) becomes

$$\operatorname{Re} \left\{ \frac{-\gamma - 1}{1 + \lambda z} + \frac{1 - \lambda z}{1 + \lambda z} \right\} \geq -\frac{1}{2} - \frac{\beta + \gamma}{2}, \quad z \in U$$

and since

$$\operatorname{Re} \left\{ \frac{-\gamma - 1}{1 + \lambda z} + \frac{1 - \lambda z}{1 + \lambda z} \right\} > \frac{-\gamma - 1}{1 - |\lambda|} + \frac{1 - |\lambda|}{1 + |\lambda|}, \quad z \in U,$$

by using Theorem 2 we have:

Corollary 2. Let $\beta, \gamma \in \mathbf{R}$ with $\gamma \geq 0$ and $\beta + \gamma \geq 1$ and let $\lambda \in \mathbf{C}$ such that

$$\frac{-\gamma-1}{1-|\lambda|} + \frac{1-|\lambda|}{1+|\lambda|} \geq -\frac{1}{2} - \frac{\beta+\gamma}{2}$$

and

$$|\lambda| \leq \frac{\beta-\gamma}{\beta+\gamma}.$$

If $f \in H(U)$ with $f(0) = 0$, $f'(0) \neq 0$ and $\beta \frac{zf'(z)}{f(z)} + \frac{\gamma-\lambda z}{1+\lambda z} \prec R_{\beta+\gamma}(z)$ satisfies

$$\operatorname{Re} J(\beta, f)(z) > \frac{\beta+\gamma-1}{2} - \frac{\gamma}{1+|\lambda|}, \quad z \in U$$

then $F = A_{(\beta, \gamma), \frac{z}{1+\lambda z}}(f)$ is univalent in U ; in addition f is also univalent in U .

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(Received: 10.2.1998.)

(Revised: 8.6.1998.)